

Analysing step-drawdown tests in heterogeneous aquifers

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Abstract

All existing methods for analysing step-drawdown test data assume that the aquifer is homogeneous, whereas most real aquifers are heterogeneous. Failure to account for such heterogeneity can lead to erroneous mis-interpretation of step-drawdown test results, which may in turn result in expensive over- or under-design of long-term pumping plant. This paper reports a new method for analysing step-drawdown data which not only takes heterogeneity into account, but which also turns out to be simpler and more robust than existing methods even where the aquifer is relatively homogeneous. The new method is equally applicable to confined, leaky, and unconfined aquifers. The heart of this method is a simple data transformation procedure (which we term 'homogenization of drawdown records'), based on an obvious logic, that unambiguously separates aquifer losses and well losses. As such, the new method directly calculates the well loss in the pumping well without recourse to the equations and coefficients upon which the existing methods of analysis are based. Extending the same logic, it is apparent that well loss can also be quantified using data from constant-rate pumping tests. The new method not only yields more consistent and reliable results than existing methods, but also enhances conceptual understanding of drawdown processes and is sufficiently simple that it can be implemented rapidly by manual calculations, allowing analysis of test data in the field, even while pumping continues.

Keywords: aquifers, pumping, water wells, limestone

Step-drawdown tests are one of the most widely-used means of evaluating well performance, for they yield information about the magnitude of pumping-induced hydraulic head losses in production wells which is very useful in tasks such as the specification of permanent pumping plant and the periodic re-appraisal of well efficiency (e.g. Clark 1977, 1992). In a step-drawdown test, the well is initially pumped at a low rate until the drawdown within the well stabilizes. The discharge is then abruptly increased, and water levels monitored until drawdown stabilizes again. This process is typically repeated three or four times (i.e. for 3 or 4 'steps') until the well is perceived to be yielding water at a rate close

to or exceeding its maximum potential yield (Kruseman & de Ridder 1994; Todd 1980). Clark (1992) recommends that at least four steps should be included in a given step-test in order to obtain a reliable relationship between 'steady' drawdown and pumping rate, which defines the 'specific drawdown' function of the well, i.e. the amount of drawdown anticipated per unit discharge rate. Specific drawdown values are useful in designing permanent pumping arrangements, and also serve as a good guide to the maximum yield which a well as an engineered structure will yield (e.g. Brassington 1998): specific drawdown values will be more-or-less constant between all steps until the maximum sustainable yield of the well is exceeded, beyond which the specific drawdown value will increase rapidly (non-linearly) between successive steps as pumping rate is increased. (It ought to be pointed out that step-tests therefore test the ability of a real well to yield water, rather than the ability of the aquifer at that point to yield water were it tapped by a well with an efficiency of 100%).

Analysis of step-test data is also commonly undertaken to quantify 'well loss', which may be defined as the component of total drawdown attributable to head losses within the engineered environment of the well itself, rather than in the surrounding aquifer. As well as facilitating the rational design of pump suspension depths etc, well loss values are also extremely useful in aquifer modelling exercises, since they allow observed water levels in production wells to be compared on a like-for-like basis with numerically simulated heads, which reflect drawdowns in the aquifer itself (e.g. Lerner 1989; Younger 1990). A key problem with existing methods of calculating well loss from step-drawdown test data is that they assume the aquifer to be homogeneous, whereas most real aquifers are heterogeneous. (For instance, both of the major public-supply aquifers of England, i.e. the Chalk and the Permo-Triassic Sandstones, are famously heterogeneous). Failure to account for such heterogeneity can lead to mis-interpretation of step-drawdown test results, which may in turn result in expensive over- or under-design of long-term pumping plant, and/or failure to assess the true degree of consistency between observed and modelled groundwater heads in the vicinity of wellfields. This paper reports a new method for analysing step-drawdown data which not only takes heterogeneity into account, but which also turns out to be simpler and more robust than existing methods of analysis, even

where the aquifer is relatively homogeneous. Before detailing the new method, it is necessary to review existing methods of analysis, so that the theoretical basis for the proposed technique can be fully appreciated.

Existing methods for analysing step-drawdown test data

Jacob (1947) was the first to provide an interpretative framework for step-drawdown tests. To explain the drawdown (s_w) in the pumped well, Jacob (1947) proposed the following equation:

$$s_w = BQ + CQ^2 \quad (1)$$

where the terms BQ and CQ^2 represent the aquifer loss and the well loss respectively. (Q is the pumping rate and B and C are termed the 'aquifer loss coefficient' and the 'well loss coefficient' respectively). Subsequently, Rorabaugh (1953) proposed that well loss may not be a simple quadratic function of pumping rate, and he therefore slightly rewrote Jacob's Equation (1 above) as follows:

$$s_w = BQ + CQ^p \quad (2)$$

Where p is the power to which the pumping rate must be raised to calculate the true well loss.

Existing step-drawdown test analysis methods commonly involve determination of B , C , and p , though different authors have reported markedly variable values for p . For instance, Rorabaugh (1953) reported that p values generally fall around 2.5, except in the case of low discharge pumping tests. In contrast, Bierschenk & Wilson (1961, as cited by Kawecki 1995) and Clark (1977) found that p usually equals 2, re-vindicating the earlier suggestion of Jacob (1947). Assuming p to equal 2, Bierschenk (1963) and Hantush (1964) independently derived the same, simple, graphical method which is still widely used to determine B and C . This graphical method is based on the equation of Jacob (i.e. Equation 1 above) and can be applied in confined, leaky, and unconfined aquifers. Subsequently, Lennox (1966) argued that the most appropriate method for analysing step-drawdown test data was, after all, the Rorabaugh (1953) method, on the following grounds:

- it considers ideal flow conditions, and
- it is consistent with p values > 2 (Lennox 1966, had obtained p values around 3.5 in his analyses).

Sheahan (1971) presented a curve-fitting method to assess B , C , and p in Rorabaugh's equation (i.e. Equation 2 above). Essentially, the type-curve of Sheahan (1971) simplifies the application of the unaltered Rorabaugh (1953) method. It should be noted that in order to distinguish a unique curve from Sheahan's type curve, it is necessary that the test data

span the segment of maximum curvature in the type curve; hence the range of pumping rates covered in the step-test should be as broad as possible.

Eden & Hazel (1973) proposed a method based on the Cooper & Jacob (1946) straight-line approximation to the Theis (1935) equation for non-steady radial flow to a well. They pointed out that where the aquifer is heterogeneous or bounded, the method is not rigorously applicable. Kaergaard (1982) reviewed the physical background for Equations (1) and (2) and concluded that the results obtained using the Rorabaugh (1953) method are unsatisfactory. Yeh (1989) presented a non-graphical method based on non-linear least-squares and finite-difference approximations to calculate the aquifer loss and well loss coefficients. Avci (1992) tested all of the major methods outlined above, and found them to yield widely varied values of p and C when applied to the same data-set. Helweg (1994) pointed out that the original equation of Jacob (1947) (i.e. Equation 1) is vulnerable to criticism because it ignores the dependency of drawdown on time. Although Eden & Hazel (1973) had already overcome this shortcoming by using a robust approximation to the non-steady radial flow equation, Helweg (1994) went on to develop a computer-based method for step-test analysis (implemented in the code *DRAWDOWN*) which he termed the General Well Function (GWF). Kawecki (1995) argued that the typically wide variations in values of C and p obtained from a single data-set using the various analysis methods casts doubt on the value of using them for classificational (let alone engineering design) purposes. He therefore advocated that total well loss (rather than the well loss coefficient alone) is a more meaningful and useful indicator of well performance. Most recently van Tonder *et al.* (2001) observed heterogeneous drawdown behaviour in many pumping tests (including constant-rate tests) performed in Southern Africa. They deduced that such heterogeneous behaviour cannot be attributed only to turbulent flow effects within the boreholes, implying that aquifer heterogeneities strongly influence step-drawdown responses.

We concur with the inferences of Kawecki (1995) and van Tonder *et al.* (2001) that the large variations in the values of C and p (Equation 2) commonly reported are most likely due to the fact that existing methods of analysis systematically ignore aquifer heterogeneity, and therefore also ignore the dependency of aquifer loss on time since pumping began. The new method proposed below overcomes these defects in existing methods by explicitly accounting for time-dependent well losses and heterogeneity effects.

A simple, new method for analysing step-drawdown test data

In order to analyse step-test data, a new, simple, and direct approach has been introduced. This proposed

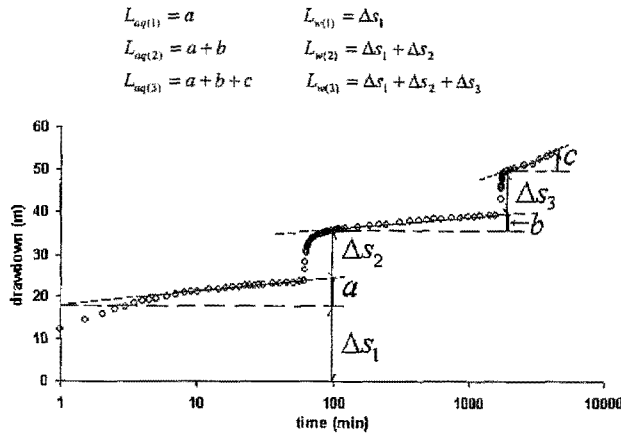


Fig. 1. Proposed method of analysing step-drawdown test data. The values of a , b and c give the values of aquifer losses for steps 1 to 3, respectively.

method does not resort to the equations proposed by Jacob (1947) and Rorabaugh (1953), and can be applied in confined, leaky and unconfined aquifers. It is equally applicable in homogeneous and heterogeneous aquifers. We first outline the method here and then explain its application in greater detail. (It should be noted that this new method has been successfully tested by the authors using a large number of data-sets from heterogeneous limestone aquifers in the UK and Ireland; however, in the interests of brevity, full results of this programme of testing are reserved to a subsequent paper).

Since total drawdown in a pumping well comprises aquifer loss plus well loss, the difference between the observed drawdown and the aquifer loss quantifies the well loss (Kawecky 1995). Using this simple logic, well loss can directly be calculated using step-test data. To this end, after stabilization of drawdown rate, the contribution of aquifer loss is separated from observed drawdown as shown in Figure 1. Therefore, the value of well loss can easily be calculated using the remaining components of drawdown (Δs_i), involving the principle of superposition. (For further details on practicalities see the section 'Procedure for calculating aquifer loss and well loss' below).

According to the Theis (1935) non-steady radial flow equations ($s = \frac{Q}{4\pi T} W(u)$ and $u = \frac{r^2 S}{4Tt}$) it is evident that at a specific time and a given radial distance from the pumping well, the theoretical drawdown (or aquifer loss) depends only on pumping rate. To calculate aquifer loss for a step-drawdown test, therefore, all values of the drawdown need to be 'homogenized', that is standardized in relation to the pumping rate of the first-step (the manner in which this is achieved is described in the next section). Using homogenized data and simple algebraic calculations, the values of the aquifer loss can be calculated for any pumping period.

Jacob (1947) has demonstrated that the term BQ is the theoretical drawdown ('formation loss') which conforms

to a straight line on a semi-log plot. In our newly proposed method, before the rate of change in drawdown stabilizes it includes both BQ and CQ^2 , whereas after stabilization of the rate of change in drawdown (i.e. the time at which the data begin to conform to a straight-line on a semi-log plot), it reflects the contribution of BQ (theoretical drawdown) only. Hence, it is clear that division of the total drawdown into well loss and aquifer loss by the proposed method is equivalent to the terms of the same name in the conventional Jacob method.

There are two reasons for homogenizing the step-drawdown test data based on one reference pumping rate: (1) distinguishing heterogeneity effects and (2) providing a generalized equation which allows calculating the values of aquifer loss at any time for any pumping rate. In heterogeneous aquifers, before homogenizing the drawdown, the discrepancies between rate of drawdown for different steps have to be attributed to both the effect of pumping rate and the effect of heterogeneity. However, after homogenizing the drawdown data based on one pumping rate, the contribution of pumping rate is omitted and the existing differences are thus attributed to the effect of heterogeneity only. In order to determine the value of aquifer loss for future constant-rate pumping tests at any time for any pumping rate, a generalized equation is needed which has to be based on one pumping rate only (see caption of Equation 5).

Homogenizing the step-test data using a reference pumping rate

As was pointed out in the previous section, it is easy to correct for the effect of changes in pumping rate on drawdown in the pumping well. In order to homogenize the step-drawdown test data based on a reference pumping rate, the first step is chosen as a reference. Drawdown for other steps can be homogenized using the following adjustment:

$$s_{a(t)} = \frac{[s_{obs(t)} - s_{obs(t_{ref})}]}{(Q_{obs}/Q_{ref})} + s_{a(t_{ref})} \quad (3)$$

where

$s_{a(t)}$ = adjusted drawdown at time t

$s_{obs(t)}$ = observed drawdown at time t

$s_{obs(t_{ref})}$ = observed drawdown at time t_{ref}

$s_{a(t_{ref})}$ = adjusted drawdown at time t_{ref}

t_{ref} = start of stabilized part of each step (reference time) (i.e. the time at which the data begin to conform to a straight line on a Jacob semi-log plot)

Q_{obs} = observed pumping rate

Q_{ref} = reference pumping rate

In order to homogenize step-drawdown test data based on a reference rate, this equation is applied for all

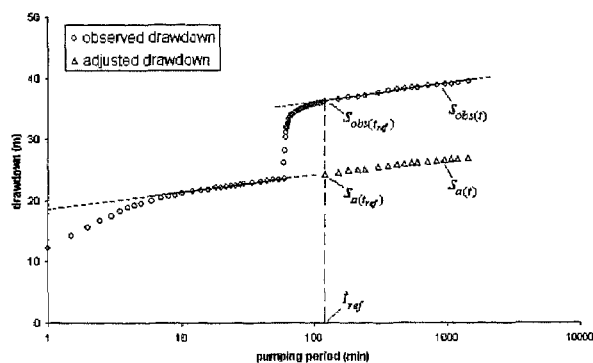


Fig. 2. Illustration of key parameters involved in Equation (3).

steps successively. Figure 2 clarifies the key parameters in Equation 3. It should be noted that this equation deals conjointly with the homogenization of both aquifer losses and well losses.

Procedure for calculating aquifer loss and well loss

(a) Calculating aquifer loss

- Homogenize the values of drawdown for all steps based on a reference pumping rate (usually the pumping rate used in the first step) as described above.
- Plot these homogenized drawdowns versus time on semi-log paper (with time on the logarithmic axis) and draw a straight line through the plotted data as shown in Figure 3a. It should be noted that for a heterogeneous aquifer the straight line should pass through each uniform segment of data as shown in Figure 3b.
- Calculate the value of aquifer loss using Equations 4 and 5 below (for homogeneous and heterogeneous aquifers, respectively):

For homogeneous aquifers:

$$L_{aq} = (m \log t) \frac{Q_{obs}}{Q_{ref}} \quad (4)$$

where

L_{aq} = aquifer loss (m)

m = slope of straight line

t = pumping time (min)

Q_{obs} = observed pumping rate

Q_{ref} = reference pumping rate

Equation 4 consists of two terms: $m \log t$ and $\frac{Q_{obs}}{Q_{ref}}$.

The first term ($m \log t$) is obviously just the equation of a straight line on a semi-log axis plot (t on logarithmic scale). The second term $\left(\frac{Q_{obs}}{Q_{ref}}\right)$ is a conversion factor which is used to transform the values of drawdown from the reference pumping rate into the target pumping rate (Q_{obs}). This factor was introduced in Equation 3, in which for homogenizing the step-drawdown test data based on a reference pumping rate, all observed drawdowns after subtracting the values of drawdown increments (which are due to well losses) were divided by this coefficient $\left(\frac{Q_{obs}}{Q_{ref}}\right)$.

For heterogeneous aquifers:

$$L_{aq} = \left(\sum_{i=1}^n m_i \log \frac{t_i}{t_{i-1}} \right) \frac{Q_{obs}}{Q_{ref}} \quad (5)$$

where

m_i = slope of straight line for i^{th} portion

t_i = pumping time during i^{th} portion

Equation 5 is a generalized form of Equation 4, which can then be used to analyse real pumping test data. This generalized equation takes into account the effect of heterogeneity by incorporating different slopes (m_i), the effect of pumping period by involving t , and the effect of pumping rate by incorporating the ratio between the target pumping rate (Q_{obs}) and the reference pumping rate (Q_{ref}). Therefore, the use of this equation allows the value of aquifer loss to be determined for future pumping tests at any pumping period for any pumping rate. Since Equation 5 is the more generalized form of Equation 4, we have demonstrated its application for a real step-drawdown test (see Appendix A).

It should be noted that the maximum value of t_i is the total pumping duration. However, for predicting the well drawdown at durations longer than the actual pumping duration, it is possible to extrapolate the

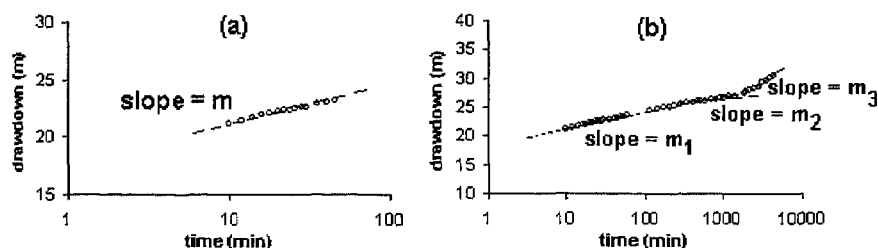


Fig. 3. Homogenized data based on the pumping rate of the first step. Examples for: (a) a relatively homogeneous aquifer and (b) a heterogeneous aquifer.

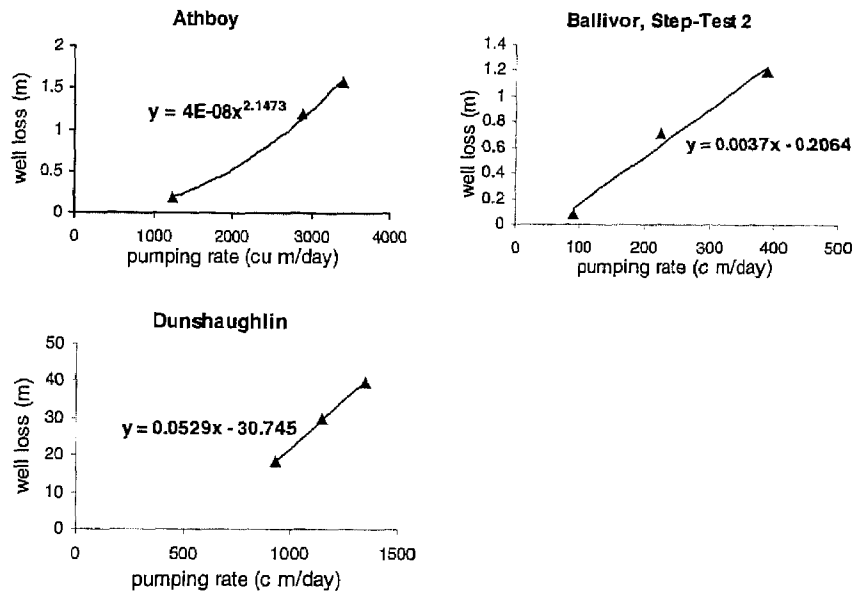


Fig. 4. Relationships between well loss and pumping rate for three production wells in Carboniferous Limestone aquifers in the Republic of Ireland (data provided by the Geological Survey of Ireland).

last portion of the homogenized data plot. For more information about calculating the aquifer loss using Equation 5 see Appendix A which gives a worked example.

(b) Calculating well loss

- For all steps, draw a best fit line through the homogeneous portion of each step using semi-log paper (time on logarithmic axis).
- Extend the straight line drawn through the homogeneous portion of the first step until it intercepts the drawdown axis where $\log t = 1$, and read the value of drawdown at the interception point; this equals the well loss for the first step.
- Determine the drawdown increments between successive steps, taken from the beginning of each step (homogeneous portion) and the drawdown for the preceding step at the same time. The latter is obtained by extrapolation using the homogeneous portion of the preceding step (Fig. 1). The drawdown increments between successive steps give the changes of well losses between them. Figure 1 shows these increments for each step. Applying information displayed on this figure and the principle of superposition the value of well losses for each step can be calculated as follows:
 - Well loss at 1st step = increment 1 = Δs_1
 - Well loss at 2nd step = increment 1 + increment 2 = $\Delta s_1 + \Delta s_2$
 - Well loss at nth step = increment 1 + increment 2 + ... + increment n = $\Delta s_1 + \Delta s_2 + \dots + \Delta s_n$
- Now plot the values of well loss for each step versus the corresponding pumping rate and then draw

the best curve through these data as shown in Figure 4. The equation of this curve can easily be determined.

- Finally, for pumping rates other than those used in the test, the well loss can be calculated either by using this equation or by interpolating the obtained curve.

We have demonstrated the application of proposed method for a real step-drawdown test and compared its results with those from conventional methods (see Appendix A).

Comparing calculated drawdown and observed drawdown

To facilitate inter-comparison between the pre-existing analytical approaches and the newly proposed method, the values of drawdown for various pumping periods for three step-drawdown test data-sets from limestone aquifers of Ireland were determined. Figure 5 shows the drawdowns calculated using the different methods and the observed drawdowns for these sites. It can be seen from the Figure 5 that the drawdowns calculated using existing the methods of step-test analysis exhibit considerable differences from the observed drawdowns. In contrast, the new method shows excellent correspondence between observed and predicted results. Since the differences between the drawdowns calculated using these common methods and observed drawdowns vary for different sites, it is difficult to compare the reliability of these methods. For example the differences between the drawdowns obtained using the Rorabaugh (1953) and the observed drawdowns for Athboy and Ballivor

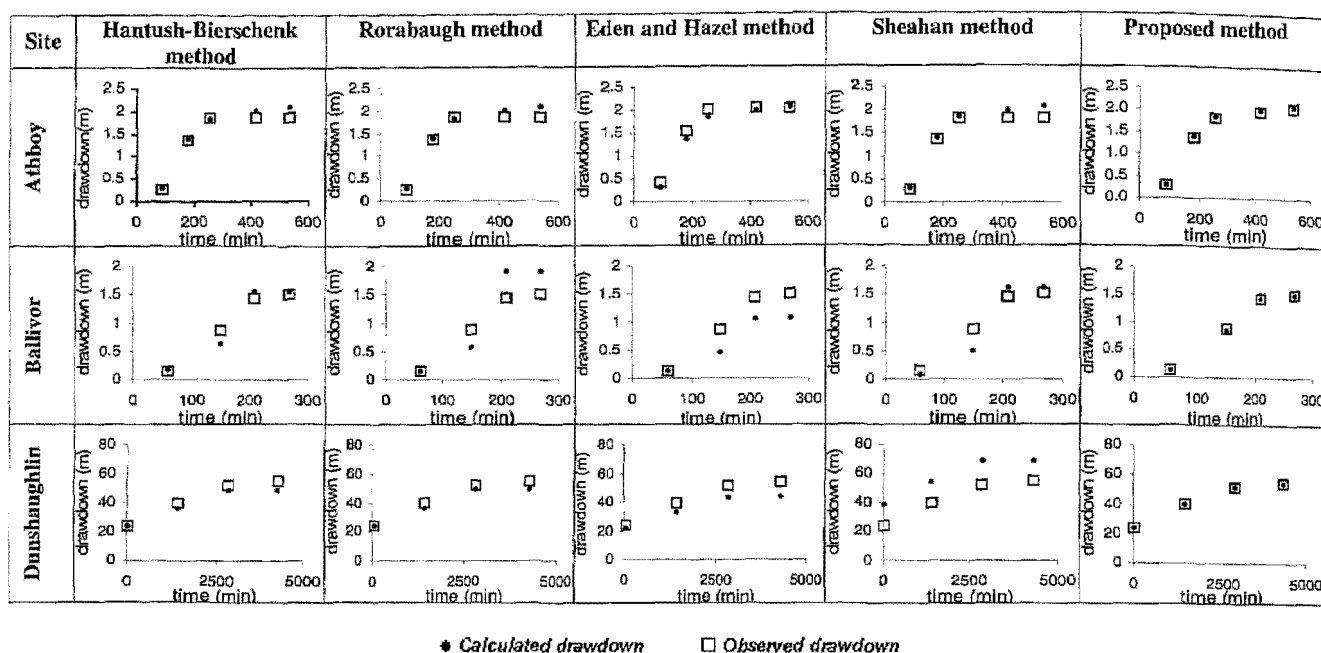


Fig. 5. Observed and calculated drawdowns using some common methods and the proposed method for three sites.

step-drawdown tests are considerably different. Figure 5 makes it clear that the new method does not suffer from this shortcoming.

Conclusions

Ignoring the dependency of aquifer loss on pumping time and/or ignoring aquifer heterogeneity effects results in large variations in the values of well drawdown obtained using existing step-test analysis methods. A new method has been devised to analyse step-drawdown test data for both homogeneous and heterogeneous aquifers which considers the dependency of aquifer loss on time and also on heterogeneity effects. This method homogenizes the step-drawdown test data based on a reference pumping rate and can be applied to confined, leaky, and unconfined aquifers. Since the proposed method directly calculates the well loss and aquifer loss yields the results with a high accuracy and can be easily implemented using manual calculations, thus fitting it for use in the field while a step-test is still underway.

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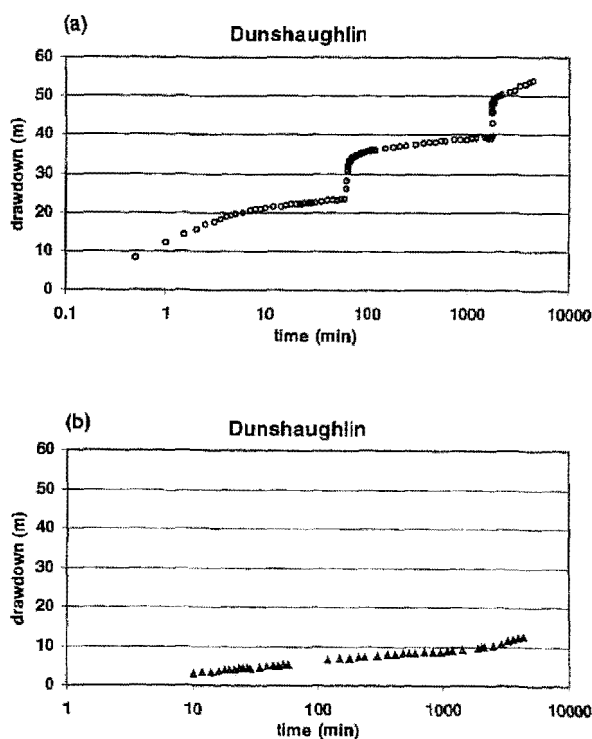


Fig. 6. Step-drawdown test data for Dunshaughlin well in Carboniferous Limestone aquifer in the Republic of Ireland. (a) step-test data before homogenizing and (b) step-test data after homogenizing.

not to be construed as representing those of any of the organisations named above.

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Appendix A

A worked example:

Table 1 (columns t and s_{obs}) and Figure 6a shows some step-drawdown test data. The following information can be derived using these step test data:

Step	Pumping rate (m^3/d)	Stabilization time (min)
1	930	10
2	1145	120
3	1340	1920

(a) Calculating aquifer loss

- By applying the Equation 3, this step test is homogenized based on a reference pumping rate (i.e. 930 m^3/d). Figure 6b gives the values of homogenized drawdown based on 930 m^3/d on a semi-log scale.
- The slopes of all segments are calculated using homogenized values of drawdown:

Duration (min)	Slope
1 – 120	2.9887
120 – 1920	2.392
1920 – 2880	6.9058
2880 – 4320	9.7298

Now, the value of aquifer loss (L_{aq}) for future constant-rate pumping tests at this site can be calculated for any pumping rate using Equation 5 as follows:

$$L_{aq} = \left[\Sigma \left(2.9887 \times \log \frac{120}{1} \right) + \left(2.3920 \times \log \frac{1920}{120} \right) + \left(6.9058 \times \log \frac{2880}{1920} \right) + \left(9.7298 \times \log \frac{4320}{2880} \right) \right] \frac{Q_{obs}}{930} \quad (6)$$

N.B. Any pumping rate can be substituted instead of Q_{obs} .

(b) Calculating well loss

- The drawdown increments between successive steps are calculated, taken from the beginning of each step (homogeneous portion) and the drawdown for the preceding step at the same time. The latter is obtained by extrapolation using the homogeneous portion of the preceding step. Table 2 gives details of calculating the drawdown increments and dwell loss for each step.
- The values of well loss for each step versus the corresponding pumping rate are plotted and then the best curve through these data drawn as shown in Figure 4 (Dunshaughlin site). The well loss equation

Table 1. Step-drawdown test data for the Dunshaughlin borehole in Carboniferous Limestone aquifer in the Republic of Ireland. Columns s_{obs} and s_a give the observed drawdowns and the adjusted drawdowns based on one pumping rate, respectively.

Step 1 ($Q = 930 \text{ m}^3/\text{d}$)			Step 2 ($Q = 1145 \text{ m}^3/\text{d}$)			Step 3 ($Q = 1340 \text{ m}^3/\text{d}$)		
$t(\text{min})$	$s_{obs}(\text{m})$	$s_a(\text{m})$	$t(\text{min})$	$s_{obs}(\text{m})$	$s_a(\text{m})$	$t(\text{min})$	$s_{obs}(\text{m})$	$s_a(\text{m})$
0	0	-	60.5	26.18	-	1741	39.70	-
0.5	8.45	-	61	28.10	-	1742	43.00	-
1	12.22	-	62	30.40	-	1743	45.60	-
1.5	14.25	-	62.5	31.10	-	1744	45.72	-
2	15.70	-	63	31.70	-	1745	45.90	-
2.5	16.76	-	63.5	32.02	-	1746	45.90	-
3	17.55	-	64	32.36	-	1747	45.90	-
3.5	18.19	-	64.5	32.67	-	1748	45.93	-
4	18.90	-	65	32.89	-	1749	46.47	-
4.5	19.13	-	66	33.28	-	1752	47.72	-
5	19.49	-	67	33.55	-	1754	47.70	-
6	20.00	-	68	33.77	-	1756	47.66	-
7	20.45	-	69	33.98	-	1758	47.70	-
8	20.73	-	70	34.12	-	1760	48.10	-
9	20.95	-	72	34.36	-	1762	48.20	-
10	21.15	21.15	74	34.55	-	1764	48.30	-
12	21.47	21.47	76	34.73	-	1766	48.10	-
14	21.69	21.69	78	34.84	-	1768	47.96	-
16	21.90	21.90	80	34.97	-	1770	48.20	-
18	22.08	22.08	82	35.08	-	1775	48.20	-
20	22.21	22.21	84	35.19	-	1780	48.60	-
22	22.35	22.35	86	35.26	-	1785	48.90	-
24	22.43	22.43	88	35.34	-	1790	49.18	-
26	22.51	22.51	90	35.42	-	1795	49.33	-
28	22.60	22.60	95	35.59	-	1800	49.50	-
30	22.69	22.69	100	35.75	-	1920	49.70	27.37
35	22.87	22.87	105	35.88	-	2040	50.13	27.67
40	23.04	23.04	110	35.98	-	2160	50.42	27.87
45	23.22	23.22	115	36.07	-	2520	51.06	28.31
50	23.33	23.33	120	36.17	24.36	2880	51.50	28.62
55	23.45	23.45	150	36.56	24.68	3240	52.58	29.37
60	23.55	23.55	180	36.87	24.93	3600	53.12	29.74
			210	36.98	25.02	3960	53.70	30.15
			240	37.13	25.14	4320	54.10	30.42
			300	37.62	25.54			
			360	37.88	25.75			
			420	38.04	25.88			
			480	38.20	26.01			
			540	38.32	26.11			
			600	38.43	26.20			
			720	38.63	26.36			
			840	38.80	26.50			
			960	38.95	26.62			
			1080	39.08	26.72			
			1200	39.19	26.81			
			1440	39.40	26.98			

N.B. Boldface font s_{obs} are those which occurred before stabilization point. In other words the boldface font s_{obs} include both aquifer loss and well loss. It should be noted that the stabilization point of drawdown rate is distinguished using semi-log plot of drawdowns (see Fig. 6a).

Table 2. Different stages of calculating drawdown increments and well losses for Dunshaughlin borehole.

Step	Equation based on stabilised portion	Drawdown at some important points*	Increments (m)	Well loss (m)
1	$s=1.298 \times \ln(t)+18.272$	$s=1.298 \times \ln(1)+18.272=18.2720$	$\Delta s_1=18.2720$	$L_{w1}=\Delta s_1=18.2720$
2	$s=1.279 \times \ln(t)+30.202$	$s=1.279 \times \ln(120)+30.202=24.4862$ $s=1.279 \times \ln(120)+30.202=36.3252$	$\Delta s_2=36.3252-24.4862=11.8390$	$L_{w2}=\Delta s_1+\Delta s_2=30.1110$
3	$s=5.4443 \times \ln(t)+8.534$	$s=5.4443 \times \ln(1920)+8.534=39.8713$ $s=5.4443 \times \ln(1920)+8.534=49.6933$	$\Delta s_3=49.6933-39.8713=9.82200$	$L_{w2}=\Delta s_1+\Delta s_2+\Delta s_3=39.9330$

* Drawdown increments between successive steps, taken from the beginning of each step (homogeneous portion) and the drawdown for the preceding step at the same time. The latter is obtained by extrapolation using the homogeneous portion of the preceding step (Fig. 6a).

Table 3. Calculated and observed drawdown using some conventional methods and proposed method for the Dunshaughlin borehole data-set.

Method	Calculated drawdowns for three pumping periods (m)		
	60 min	1440 min	4320 min
Hantush-Bierschenk	23.53	35.51	48.49
Rorabaugh	23.51	35.46	48.49
Eden and Hazel	21.61	32.11	43.31
Sheahan	38.42	53.25	68.30
Proposed	23.59	39.50	53.91
Observed drawdown	23.55	39.40	54.10

for this site can easily be derived using the equation of best-fitted curve/line has as follows:

$$L_w = 0.0529 \times Q_{obs} - 30.745 \quad (7)$$

The value of well loss for any pumping rate can be calculated either by using this equation or by interpolating the obtained best fitted curve/line.

(c) Comparing calculated drawdown and observed drawdown

- The value of total drawdown for any pumping period and any pumping rate can be calculated using Equation 6 (which calculates aquifer loss) and Equation 7 (which calculates well loss). The values of drawdowns for some pumping periods for Dunshaughlin borehole were calculated and compared with observed drawdowns using the proposed method and some conventional pre-existing methods (Table 3).